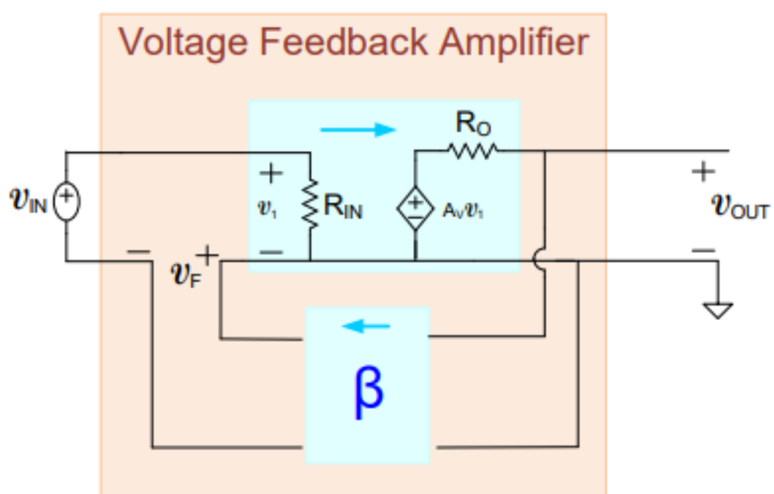
EE435 HW 1 Solution:

Problem 3 A block diagram of what is often termed a voltage-series feedback amplifier is shown below where it is assumed that the forward A amplifier is a voltage amplifier with an input impedance of R_{IN}, an output impedance of R_0 and a forward voltage gain of A_V. The feedback amplifier, denoted with the symbol β, is assumed to be an ideal voltage amplifier (likely an attenuator) with infinite input impedance and zero output impedance with feedback signal $v_F = \beta v_{OUT}$. The desensitivity, D, of a feedback amplifier is defined by the expression $D = 1 + A_V\beta$. The voltage gain A_V can itself be frequency dependent and modeled by the expression $A_V(s) = \dfrac{A_{V0}}{\frac{s}{BW}+1}$ where A_{V0} is the dc gain of the forward amplifier and BW is the bandwidth of the forward amplifier. Under these assumptions, show analytically that

a) The input impedance of the feedback amplifier is improved by D (i.e. $R_{INF}=R_{IN}D$)
b) The output impedance of the feedback amplifier is improved by D (i.e. $R_{OF}=R_O/D$)
c) The closed loop bandwidth is improved by D (i.e. $BW_{FB}=BW*D$)
d) The sensitivity of A_{FB} with respect to A_V has improved by D (i.e. $S_{A_V}^{A_{FB}} = \dfrac{S_{A_V}^{A_V}}{D}$)

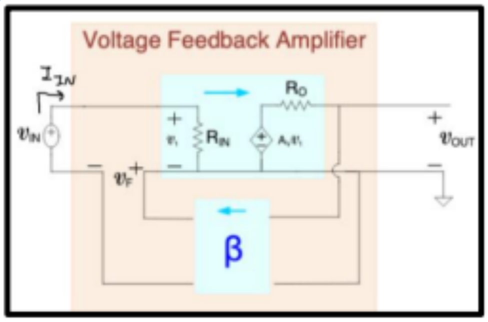


Note: The improvements in this amplifier appear to be dramatic since the desensitivity D can be very large. But the improvements might not be quite as good as the calculations suggest since the assumptions of ideality of the β amplifier may be difficult to completely meet and because it may be difficult to get perfect summing of V_{IN} and V_F. But in good feedback designs, the improvements in these parameters can be really significant.

Problem 3 Solutions

Part A

To begin, define a new current (I_{IN}) which is the current leaving the input voltage source on the left side of the feedback amplifier.



We can express the current I_{IN} in terms of the voltages and resistances which are present in the circuit:

$$I_{IN} = \frac{V_{IN} - V_F}{R_{IN}}$$

It is given that $V_F = \beta V_{OUT} = \beta A_V V_1$. We can rework our expression for I_{IN} slightly:

$$I_{IN} = \frac{V_{IN} - \beta A_V V_1}{R_{IN}}$$

Note that we have voltages on the right side of the expression, currents on the left, and that resistance is equal to voltage divided by current. Our goal is to find resistance, so let's divide by current:

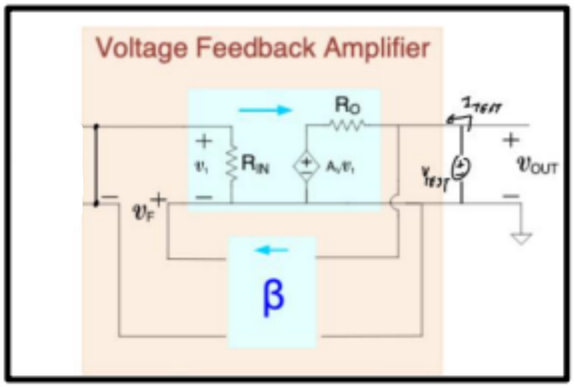
$$R_{IN} = \frac{V_{IN} - \beta A_V V_1}{I_{IN}} = \frac{V_{IN}}{I_{IN}} - \frac{\beta A_V V_1}{I_{IN}}$$

V_{IN}/I_{IN} is equal to R_{INF}. V_1/I_{In} is equal to R_{IN}. So;

$$R_{IN} = R_{INF} - \beta A_V R_{IN} \rightarrow R_{IN} + \beta A_V R_{IN} = R_{INF}$$
$$R_{INF} = R_{IN}(1 + \beta A_V) = R_{IN} D$$
$$\therefore R_{INF} = R_{IN} D$$

Part B

To find R_{OUTF}, we can follow a similar process as in Part A. We'll apply a test voltage to the output and measure the resulting test current. Additionally, we will short-circuit the input voltage source that way we do not see the effects of any voltage gained in our calculations.



Let's define I_{TEST} in terms of the existing voltages and resistances:

$$I_{TEST} = \frac{V_{TEST} - A_V V_1}{R_{OUT}} = \frac{V_{TEST} + A_V V_F}{R_{OUT}} = \frac{V_{TEST} + A_V \beta V_{OUT}}{R_{OUT}}$$

We previously shorted V_{OUT} to V_{TEST}. So:

$$I_{TEST} = \frac{V_{TEST} + A_V \beta V_{TEST}}{R_{OUT}}$$

$$R_{OUT} = \frac{V_{TEST} D}{I_{TEST}} = R_{OUTF} D$$

$$R_{OUTF} = \frac{R_{OUT}}{D}$$

Part C

Recall that the gain of the forward amplifier, A_V, is given as follows:

$$A_V(s) = \frac{A_{V0}}{\frac{s}{BW} + 1}$$

When placed in feedback the overall amplifier gain, A_{CL}, is given as follows:

$$A_{CL}(s) = \frac{A_V(s)}{1 + \beta A_V(s)}$$

Substituting $A_V(s)$ in allows us to find an expression for the overall amplifier gain in terms of BW, A_{V0}, and β:

$$A_{CL}(s) = \frac{\frac{A_{V0}}{\frac{s}{BW} + 1}}{1 + \frac{\beta A_{V0}}{\frac{s}{BW} + 1}} = \frac{A_{V0}}{\frac{s}{BW} + 1 + \beta A_{V0}}$$

With some simple manipulation:

$$A_{CL}(s) = \frac{\frac{A_{V0}}{1 + \beta A_{V0}}}{\frac{s}{BW(1 + \beta A_{V0})} + 1} = \frac{\frac{A_{V0}}{D}}{\frac{s}{BW * D} + 1}$$

See that not only has the DC gain of the closed loop amplifier changed, but the amplifier bandwidth has increased by a factor of D. Now, $BW_{FB} = BW * D$.

Part D

The sensitivity of some variable y with respect to another variable x (e.g. S_x^y) is defined as follows:

$$S_x^y = \frac{x}{y}\frac{\partial y}{\partial x}$$

First, let's calculate $S_{A_V}^{A_V}$:

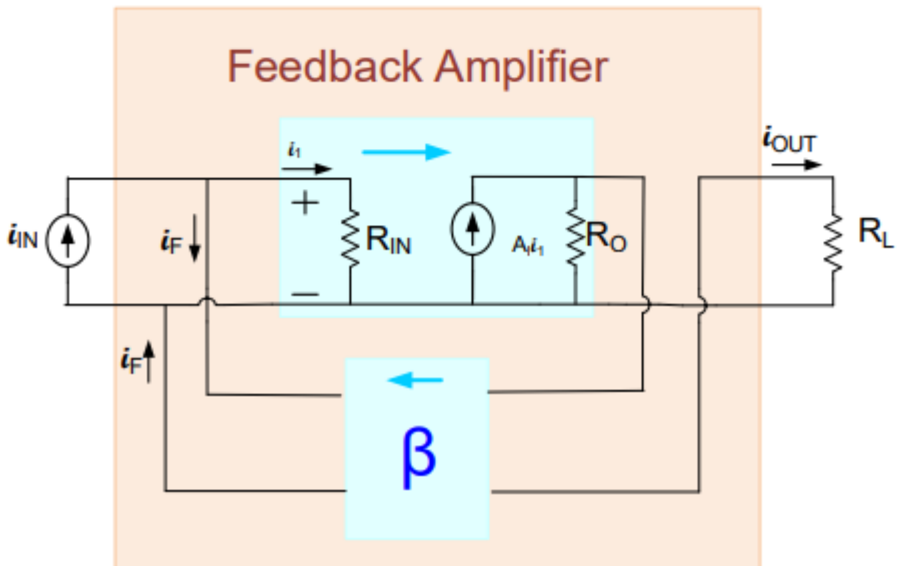
$$S_{A_V}^{A_V} = \frac{A_V}{A_V}\frac{\partial A_V}{\partial A_V} = 1$$

Now, let's calculate $S_{A_V}^{A_{FB}}$:

$$S_{A_V}^{A_{FB}} = \frac{A_V}{A_{FB}}\frac{\partial A_{FB}}{\partial A_V} = \frac{A_V}{A_{FB}} * \frac{1}{(1 + \beta A_V)^2} = \frac{A_V(1 + \beta A_V)}{A_V}\frac{1}{(1 + \beta A_V)^2} = \frac{1}{1 + \beta A_V} = \frac{1}{D} * 1$$

$$\therefore S_{A_V}^{A_{FB}} = \frac{1}{2} S_{A_V}^{A_V}$$

Problem 4 In the previous problem, feedback was used to build a voltage feedback amplifier with improvements in four key characteristics. Consider now the current feedback amplifier shown below where it is assumed that the β amplifier is an ideal current amplifier with zero input impedance and infinite output impedance characterized by the equation $i_F = \beta i_{OUT}$. The desensitivity is now defined by the expression $D = 1 + A_I\beta$. Express R_{INF} in terms of the desensitivity and the parameters in the feedback amplifier and comment on these results relative to what was obtained for the voltage feedback amplifier in the previous problem.



Problem 4 Solutions

Use a similar process as in Problem 3, Part A:

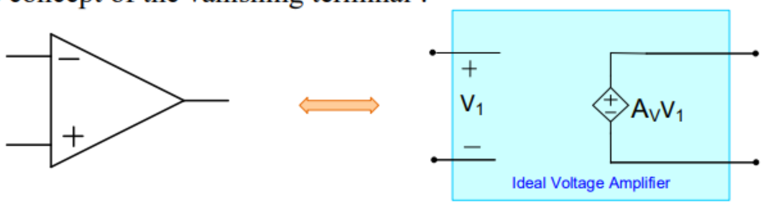
$$V_{INF} = i_1 R_{IN} = R_{IN}[I_{INF} - I_F] = R_{IN}[I_{INF} - \beta I_{OUT}]$$

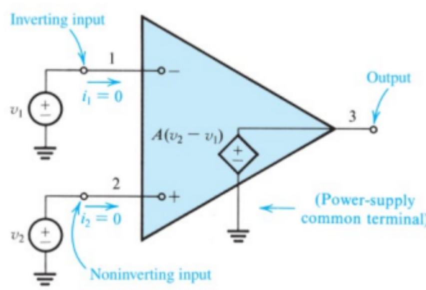
$$R_{IN} = \frac{V_{INF}}{I_{INF} - \beta I_{OUT}}$$

$$\frac{1}{R_{IN}} = \frac{I_{INF} - \beta I_{OUT}}{V_{INF}} = \frac{1}{R_{INF}} - \frac{\beta I_{OUT}}{V_{INF}} = \frac{1}{R_{INF}} - \frac{\beta A_I i_1}{i_1 R_{IN}}$$

$$R_{INF} = \frac{R_{IN}}{1 + \beta A_I} = \frac{R_{IN}}{D}$$

Problem 5 In most texts, data sheets, and circuit schematics the operational amplifier is represented as a 3-terminal device yet the two-port model of an operational amplifier has 4 terminals (i.e. nodes) as shown below. It thus appears that one terminal has somehow vanished in the symbol or equivalently appeared in the two-port model. Correspondingly, the model of the operational amplifier that is described in the most recent version of the Sedra-Smith text shows four terminals (one designated with the ground symbol) yet this ground terminal does not appear to be on the op amp symbol or on the pinout of the operational amplifiers we are using in the laboratory. Rigorously reconcile this concept of the vanishing terminal !

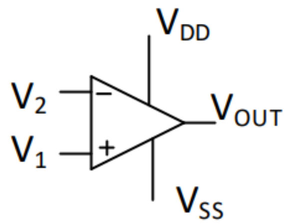




Problem 5 Solution

The V_{DD} and V_{SS} terminals are at the AC ground and thus either terminal can be considered as the missing terminal. The $A(V_2 - V_1)$ source is the small signal voltage on the output terminal.

Problem 6 The actual op amp is a five-terminal device. After reconciling this issue of the vanishing terminal in the previous problem, give an expression for the nodal output voltage with respect to V_{SS} for the operational amplifier shown below which explicitly shows the 5 terminals of the device.



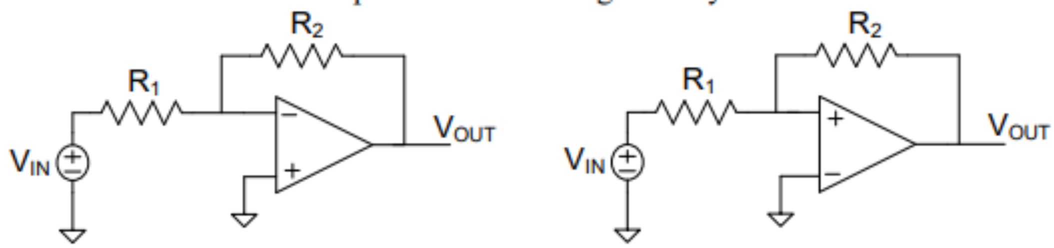
Problem 6 Solution

$$V_{OUT} = V_{OUTQ} + A_v(V_1 - V_2)$$ or, equivalently

$$V_{OUT} = V_{SS} + V_Q + A(V_1 - V_2)$$ where V_Q is the quiescent branch voltage between the V_{OUT} terminal and the V_{SS} terminal.

Problem 7 Two circuits that use a single operational amplifier are shown below.

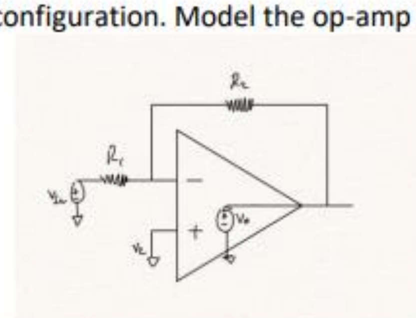
a) Using the model of the standard model of the operational amplifier that appears in the Sedra/Smith book, analyze the two circuits under the assumption that the voltage gain A_V is finite.
b) Compare the voltage gain of the two circuits as the voltage gain A_V goes to ∞
c) Although an engineer should be able to analyze any interconnection of basic devices and components, almost all basic electronics textbooks are silent on the existence of the simple circuit on the right. Why is this circuit seldom discussed?



Problem 7 Solutions

Part A

Begin with the inverting (left) configuration. Model the op-amp as follows:



Let the negative input terminal be V_{INN}. We can express the output of the op-amp as $-V_{INN}A_V$ or, equivalently, can express V_{INN} as $-\frac{V_{OUT}}{A}$. If we choose the second option, we can solve as normal:

$$\frac{-\frac{V_{OUT}}{A} - V_{IN}}{R_1} + \frac{-\frac{V_{OUT}}{A} - V_{OUT}}{R_2} = 0$$

$$V_{OUT}\left[\frac{-R_2}{A} - \frac{R_1}{A} - R_1\right] + V_{IN}[-R_2] = 0$$

$$\frac{V_{OUT}}{V_{IN}} = A_{CL} = -\frac{R_2}{R_1 + \frac{R_1}{A} + \frac{R_2}{A}}$$

Doing the same with the noninverting configuration yields the following:

$$\frac{V_{OUT}}{V_{IN}} = \frac{R_2}{-R_1 + \frac{R_2}{A} + \frac{R_1}{A}}$$

Part B

As A approaches ∞, both gains approach $-R_2/R_1$.

Part C

The second circuit is not typically used as an amplifier because its transfer function will contain a pole in the right-half plane under the correct circumstances. To see this, observe the closed-loop response of the amplifier:

$$A_{FB} = \frac{A(s)}{1-\beta A(s)} = \frac{\frac{A_{V0}}{\frac{s}{BW}+1}}{1-\frac{\beta A_{V0}}{\frac{s}{BW}+1}} = \frac{A_{V0}}{\frac{s}{BW}+1-\beta A_{V0}} = \frac{\frac{A_{V0}}{1-\beta A_{V0}}}{\frac{s}{BW(1-\beta A_{V0})}+1}$$

A pole is any frequency at which the denominator of a transfer function goes to zero. Solve the denominator for zero:

$$\frac{s}{BW(1-\beta A_{V0})}+1=0$$
$$s=-BW(1-\beta A_{V0})$$

Here, we see that some **positive** value of s can cause the denominator to be equal to 0, meaning a RHP pole exists. As an example, let's consider a special case where $A_{V0}=21$ and $\beta=\frac{1}{10}$. Here, the pole location is $-BW(1-2.1)=1.1*BW$, which is a right half plane pole.